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REGULAR ARTICLE

The value of Electronic Marketplace in a perishable product inventory system with auto-correlated demand

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Abstract This paper studies a periodic review inventory system for perishable products with correlated demand on a finite horizon. In this system, a fixed quantity of products from the supplier is received in each period. This quantity must be determined before the first period and cannot be changed thereafter. The inventory level can be adjusted through purchasing and selling products in an electronic marketplace at the beginning of each period. The available supply and demand quantities in the electronic marketplace depend on the prices offered by the retailer. The retailer's optimal purchasing and selling quantities, and respective prices in the electronic marketplace are computed, and the expected total cost is shown to be convex with respect to the order quantity from the supplier, which enables an efficient algorithm in obtaining the optimal order quantity. Numerical experiments show that greater cost savings from electronic marketplace are obtained when demands in different periods are strongly correlated and greatly differ from each other.

Keywords Inventory control · Electronic marketplaces · Auto-correlated demand process

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1 Introduction

This paper investigates a periodic review inventory system for perishable products in the presence of an electronic marketplace (EM). Examples of perishable products include vegetable, medicine, fashion clothes, etc. Nowadays, many high-tech products can also be regarded as perishable products because they usually have a short product lifetime with rapid depreciation. One way to improve inventory control for perishable products is to employ EMs as ‘spot market’, which can be alternative suppliers as well as places to salvage excess inventory.

The classical Newsboy problem is a well-recognized inventory control problem for perishable products with a single period lifetime. When products have a lifetime of longer than one period, the inventory may be depleted in the sequence of FIFO (first in first out) or LIFO (last in first out). Nahmias (1982) provided a comprehensive literature review of the inventory system for perishable products under the FIFO assumption, and Cohen and Pekelman (1978) investigated inventory control problems of perishable products under the LIFO assumption. As dynamic pricing can be easily implemented with the help of advanced information technologies, many researchers have considered dynamic pricing in inventory systems for perishable products in recent years. A comprehensive literature review for dynamic pricing of inventory systems is given by Elmaghraby and Keskinocak (2003). This paper differs from the previous work in that we focus on how to reduce the inventory cost through an EM as ‘spot market’.

Because EMs are not restricted by time and space, they connect more companies, thus enriching buyers‘ procurement choices and bringing more customers to sellers. An EM can supply emergency orders while absorbing excess inventories. By coordinating EMs with their current supply chains, companies may significantly reduce their supply chain cost. This has aroused great interests from both industrial and academic communities. Keskinocak and Tayur (2001) provided a comprehensive analysis on the deployment of EMs to streamline supply chain management. A good literature review of EMs is found in Grieger (2003). Similarly, comprehensive literature reviews on the supply chain management for E-Business are provided by Geoffrion and Krishnan (2003a,b) and Simchi-Levi et al. (2004).

The EMs provide companies ‘spot sourcing’ opportunities, where transactions are carried out on a real-time basis. This makes EMs distinguished from conventional suppliers. In the inventory system studied in this paper, a fixed quantity of products is replenished from a supplier at every period. The quantity must first be determined and cannot be changed thereafter. Purchasing a fixed quantity for a fixed time period reduces uncertainty on demand for suppliers, and the investment and the operating costs necessary to provide goods and services to customers can significantly be reduced (Grossman et al. 2000). In competitive markets, such cost savings are often passed to customers in the form of price incentives. While purchasing from EM allows the retailer to purchase the exact amount to satisfy its demand, the EM price is usually significantly higher than this "fixed quantity purchasing cost" and thus, mostly considered a secondary alternative source. At the beginning of each period, products can be purchased (sold) from (to) an EM to adjust the inventory level for the anticipated demands. The availability of supply (demand) in the EM is determined by the price offered, i.e., the higher (lower) price offered for purchasing (selling), the more supply (demand) will be available. At the end of each period, the inventory salvage and the stockout costs are computed. Unsatisfied demands are lost and the remaining inventories are salvaged. Demand is assumed to be auto-correlated in different periods.

The rest of this paper proceeds as follows. In Sect. 2, a mathematical model is developed to determine the purchasing (selling) quantity and the corresponding price to offer in the EM. The model is employed to compute the optimal order quantity from the supplier as well. Section 3 provides several optimality properties from the model developed in Sect. 2. Section 4 presents numerical results on the cost savings from the EM and analyzes the impacts of demand processes on the cost savings.

2 Mathematical model for EM and order quantity

The retailer needs to determine (1) the order quantity from the supplier and (2) bidding price and quantity in the EM. The second decision is referred to as a bidding decision in the EM.

In this system, only the relative purchasing and selling costs in the EM are computed. The relative purchasing cost is the purchasing price (in the EM) minus the unit ordering cost (from the supplier), whereas the relative selling cost equals to the unit ordering cost (from the supplier) minus the selling price (in the EM). This is because one unit of product has to be ordered from the supplier or purchased from the EM to satisfy one unit of demand. The difference between these two alternatives is completely represented by the relative purchasing cost in the EM. Similarly, when one item is sold to the EM, the associated cost is completely represented by the relative selling cost in the EM.

The following assumptions are employed to develop a mathematical model.

1. *Available supply (demand) from (to) the EM is a concave increasing function with respective to the relative purchasing (selling) cost, represented by* $\gamma(C_{p,k})$*(and* $\eta(C_{s,k})$*, respectively). Both* $\gamma(C_{p,k})$ *and* $\eta(C_{s,k})$ *are twice differentiable, and* $\gamma(0) = 0$ *and* $\eta(0) = 0$.
   We assume supply (demand) in the EM is a deterministic function of the relative purchasing (selling) price. While a stochastic function is not considered, the deterministic assumption simplifies the model to be tractable. The assumption still considers the nature of "spot market" where the higher (lower) price attracts more supplies (demands). Furthermore, we assume that the marginal cost to attract supply and demand is increasing. The same deterministic assumption is also employed by Federgruen and Heching (1999). Moreover, the deterministic function is still less restrictive than a linear or an exponential function employed in the previous studies.

2. *Unsatisfied demand in each period is lost and excessive inventories are salvaged. Thus, there is no inventory carrying forward.*
   This assumption applies to perishable products with a lifetime of one period. We are aware that this assumption may cause some loss of generality. However, since the objective of this study is to investigate how to reduce the inventory cost for perishable products with EMs, this assumption is applied to simplify the mathematical model.
3. *Demands are auto-correlated in different periods.*
4. *There is no fixed cost associated with purchasing and selling products in the EM.*

The fourth assumption is reasonable since transactions in EM are often considered as "spot sourcing," which does not involve complex negotiation processes. Thus, the fixed cost for EM transactions is negligible, compared with the purchasing cost.

The notations for the mathematical model are as follows:

Q:	Order quantity from the supplier;
$Y_{p,k}$:	Purchasing quantity from the EM at period k;
$Y_{s,k}$:	Selling quantity to the EM at period k;
$C_{p,k}$:	Relative purchasing cost in the EM at period k;
$C_{s,k}$:	Relative selling cost in the EM at period k;
$\gamma(C_{p,k})$:	Available supply from the EM at a bidding cost $C_{p,k}$;
$\eta(C_{s,k})$:	Available demand from the EM at a bidding cost $C_{s,k}$;
h:	Unit salvage cost;
π:	Unit stockout cost;
N:	Number of periods over the planning horizon;

The additional notations for the forecast of auto-correlated demand are as follows:

d_t:	Demand level in period t;
$\hat{D}_t$:	Realized demand in period t and previous periods, $\hat{D}_t = [d_t, d_{t-1}, \ldots]$;
$f_{t+s}(.\vert\hat{D}_t)$:	Probability distribution function of demand in period $t+s$, based on $\hat{D}_t$;
$F_{t+s}(.\vert\hat{D}_t)$:	Cumulative distribution function of demand in period $t+s$, based on $\hat{D}_t$;

Using the notations, the mathematical model to determine the bidding decision in the EM is developed. At the beginning of period k, the retailer determines $C_{p,k}, C_{s,k}, Y_{p,k}$ and $Y_{s,k}$, based on the updated demand distribution in period $k, f_k(.|\hat{D}_{k-1})$ and $F_k(.|\hat{D}_{k-1})$. Because unsold inventories are salvaged at the end of the period, only the expected cost in period k needs to be considered. This problem is formulated as in $P1$.

$$P1\; C_k(Q|\hat{D}_{k-1}) = \underset{C_{p,k},C_{s,k},Y_{p,k},Y_{s,k}}{Min} : C_{p,k} * Y_{p,k} + C_{s,k} * Y_{s,k} + L_k(Q + Y_{p,k} - Y_{s,k}) \tag{1}$$

$$L_k(x) = h * \int_0^x (x-\omega)^* f_k(\omega|\hat{D}_{k-1})\mathrm{d}\omega + \pi^* \int_x^{+\infty} (\omega - x) * f_k(\omega|\hat{D}_{k-1})\mathrm{d}\omega$$

Subject to: $C_{p,k}, C_{s,k} \geq 0, \quad 0 \leq Y_{p,k} \leq \gamma(C_{p,k}), \quad 0 \leq Y_{s,k} \leq \mathrm{Min}[Q, \eta(C_{s,k})]$

In Eq. (1), $C_k(Q|\hat{D}_{k-1})$ represents the minimal expected cost in period k for the given order quantity Q and the latest demand information $\hat{D}_{k-1}.C_{p,k}{}^* Y_{p,k}$ is the relative purchasing cost when $Y_{p,k}$ are purchased from the EM at a relative cost of $C_{p,k}$. Similarly, $C_{s,k} * Y_{s,k}$ is the relative selling cost when $Y_{s,k}$ are sold to the EM at a relative cost of $C_{s,k}.L_k(.)$ stands for the expected inventory holding and stockout cost in period k when the demand distribution is $f_k(.|\hat{D}_{k-1})$.

The optimal order quantity from the supplier, Q, must minimize the total expected cost, $C(Q)$. The problem is formulated as in $P2$.

$$P2\; \underset{Q\geq 0}{\mathrm{Min}} : C(Q) = \sum_{k=1}^{N} \underset{\hat{D}_{k-1}}{E} \left(C_k(Q|\hat{D}_{k-1}) \right) \tag{2}$$

In Eq. (2), $C_k(Q|\hat{D}_{k-1})$ must first be obtained by solving problem $P1$. $\underset{\hat{D}_{k-1}}{E} \left(C_k(Q|\hat{D}_{k-1}) \right)$ is the expected value of $C_k(Q|\hat{D}_{k-1})$ over $\hat{D}_{k-1}$.

3 Optimal bidding decision in the EM and order quantity from the supplier

This section provides several optimality properties of the mathematical model developed in the previous section. $P1$ must first be solved to obtain the optimal bidding decision in the EM. Then, $P2$ is solved with the solution of $P1$.

The optimal bidding decision in period k is defined by $C^*_{p,k}, C^*_{s,k}, Y^*_{p,k}$ and $Y^*_{s,k}$. Let $F_k^{-1}(.|\hat{D}_{k-1})$ stand for the inverse function of $F_k(.|\hat{D}_{k-1})$.

Lemma 1 *Let* $X_k = F_k^{-1}\left(\frac{\pi}{\pi+h}|\hat{D}_{k-1}\right)$. *When* $Q < X_k$, $Y^*_{s,k} = 0 = \eta(0) = \eta(C^*_{s,k})$ *and* $Y^*_{p,k} = \gamma(C^*_{p,k})$; *when* $Q > X_k, Y^*_{p,k} = 0 = \gamma(0) = \gamma(C^*_{p,k})$ *and* $Y^*_{s,k} = \eta(C^*_{s,k})$.

Proof X_k minimizes $L_k(x)$. When $Q < X_k$, selling products to the EM incurs a relative selling cost, and this increases the expected inventory cost. Therefore, $C^*_{s,k} = 0$ when $Q < X_k$. Similarly, $C^*_{p,k} = 0$ when $Q > X_k$. □

When $Q < X_k$, if $Y^*_{p,k} < \gamma(C^*_{p,k})$, the minimal expected cost in period k is given by Eq. (3).

$$C_k(Q|\hat{D}_{k-1}) = C^*_{p,k} * Y^*_{p,k} + L_k(Q + Y^*_{p,k}) \tag{3}$$

However, if $Y^*_{p,k} < \gamma(C^*_{p,k})$, there must exist $\hat{C}_{p,k} < C^*_{p,k}$ that satisfies $Y^*_{p,k} = \gamma(\hat{C}_{p,k})$. With $\hat{C}_{p,k}$ and $Y^*_{p,k}$, the expected cost in period k is given by Eq. (4).

$$\tilde{C}_k(Q|\hat{D}_{k-1}) = \hat{C}_{p,k} * Y^*_{p,k} + L_k(Q + Y^*_{p,k}) \tag{4}$$

Because $(\hat{C}_{p,k}, Y^*_{p,k})$ incurs a lower cost than $(C^*_{p,k}, Y^*_{p,k})$, this contradicts the assumption that $C^*_{p,k}$ and $Y^*_{p,k}$ are optimal. Therefore, $Y^*_{p,k} = \gamma(C^*_{p,k})$.

Similarly, when $Q > X_k$, $Y^*_{s,k} = \eta(C^*_{s,k})$. □

From Lemma 1, $C_k(Q|\hat{D}_{k-1})$ is computed as follows.

$$C_k(Q|\hat{D}_{k-1}) = \underset{C_{p,k}, C_{s,k}}{\mathrm{Min}} : A_k(C_{p,k}, C_{s,k}, Q|\hat{D}_{k-1}),$$

where

$$\begin{aligned} A_k(C_{p,k}, C_{s,k}, Q|\hat{D}_{k-1}) = {} & C_{p,k} * \gamma(C_{p,k}) + C_{s,k} * \eta(C_{s,k}) \\ & + L_k\big(Q + \gamma(C_{p,k}) - \eta(C_{s,k})\big) \end{aligned} \tag{5}$$

Theorem 1 (i) *If* $Q < X_k, A_k(C_{p,k}, 0, Q|\hat{D}_{k-1})$ *is convex with respect to* $C_{p,k}$; (ii) *if* $Q > X_k, A_k(0, C_{s,k}, Q|\hat{D}_{k-1})$ *is convex with respect to* $C_{s,k}$; (iii) *if* $Q = X_k, C^*_{p,k} = C^*_{s,k} = 0$.

Proof See Appendix A. □

From the convex property shown in Theorem 1, $C^*_{p,k}$ and $C^*_{s,k}$ can be obtained by an efficient search algorithm such as the golden search algorithm. Once the values of $C^*_{p,k}$ and $C^*_{s,k}$ are computed, $Y^*_{p,k}$ and $Y^*_{s,k}$ are obtained using Lemma 1 and $C_k(Q|\hat{D}_{k-1})$ is computed by Eq. (5).

Theorem 2 $C(Q)$ *is convex with respect to* Q.

Proof See Appendix B. □

Since it is difficult to obtain a closed form solution for the optimal Q, a near optimal solution can be obtained efficiently by the simulation-based search using the convex property.

4 Numerical experiment

In this section, a numerical experiment is conducted to illustrate the cost savings from the EM under various demand processes. Without the EM, the retailer only needs to determine the order quantity from the supplier. In Appendix C, a solution procedure to compute the retailer's optimal order quantity without the EM is provided.

4.1 Experiment design

The demand process is assumed to be an auto-regressive AR(1) process, as given by Eq. (6), where $-1 < \phi < 1$. ε_t follows an identical independent Normal distribution with mean 0 and variance σ^2.

$$d_{t+1} - \mu = \phi * (d_t - \mu) + \varepsilon_{t+1} \tag{6}$$

The AR(1) demand process is considered in Lee et al. (2000), Kahn (1987) and Miller (1986). In order for the numerical study, we assume that the retailer may accurately estimate the values of μ, ϕ and σ.

Equation (6) implies that for a given $\hat{D}_t$, the conditional distribution of d_{t+1} is a Normal distribution with mean $\mu + \phi * (d_t - \mu)$ and variance σ^2. In this numerical experiment, the parameters of the demand process ϕ and σ are set to different levels to represent different demand processes, where ϕ represents the correlations among demands in different periods. Different values for the demand in period 0 (the period before the first period under consideration) d_0 are also considered.

Supplies and demands in the EM are determined by $\gamma(C_p) = 10 * C_p$ and $\eta(C_s) = 10 * C_s$. The linear demand–cost function is also found in Federgruen and Heching (1999). Table 1 summarizes the values of parameters employed in this numerical experiment.

For each combination of the parameters, the experiment is replicated 100 times and the average cost per period is computed.

4.2 Impact of the demand process on the cost savings from the EM

This numerical experiment focuses on how the cost savings from the EM are affected by the parameters of the demand process. The impacts of ϕ on the cost savings is shown in Figs. 1 and 2.

Figures 1 and 2 show that the cost savings at larger $|\phi|$ values are greater than the cost savings at smaller $|\phi|$ values.

Table 1 Value of parameters in the numerical experiment

Parameters	Value					
ϕ	-0.9	-0.5	-0.1	0.1	0.5	0.9
σ	1	3	5			
d_0	50	100	150			
N	10	100				
μ	100					
h	1					
π	5					

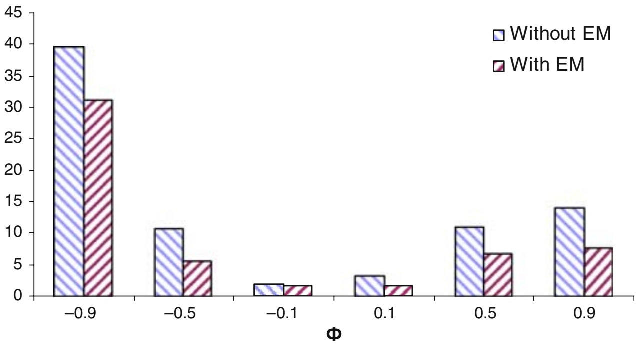

Fig. 1 Average cost per period when $N = 10, \sigma = 1, d_0 = 150$

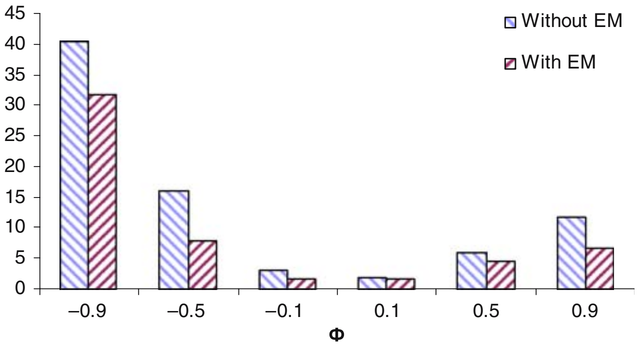

Fig. 2 Average cost per period when $N = 10, \sigma = 1, d_0 = 50$

When $|\phi|$ becomes smaller ($\phi = \pm 0.1$ in the numerical experiment), demand in each period becomes almost identically and independently distributed. Thus, the cost savings from using the EM will not be significant. However, when $|\phi|$ becomes larger ($\phi = \pm 0.9$ in the numerical experiment), demand distribution in the current period is highly dependent on demand in the previous period. This implies that the past demand can significantly affect the impending demand in the current period. The retailer then updates her demand forecast accordingly and can adjust her inventory level using the EM to reduce the inventory cost. Figures 3 and 4 provide additional experimental results to support the claim.

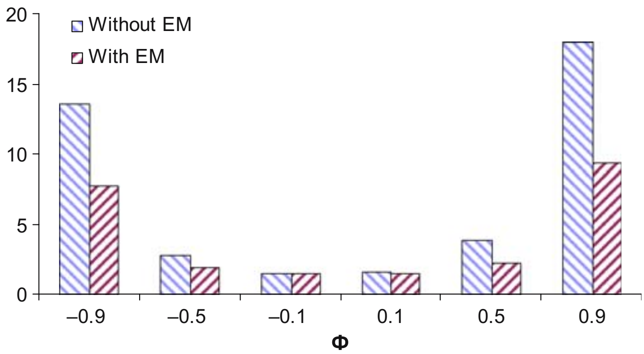

Fig. 3 Average cost per period when $N = 100, \sigma = 1, d_0 = 150$

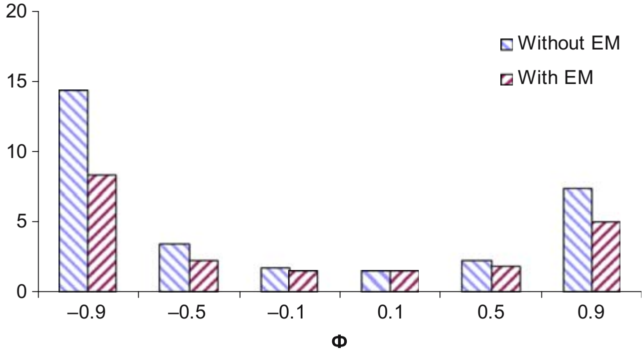

Fig. 4 Average cost per period when $N = 100, \sigma = 1, d_0 = 50$

The results shown in Figs. 1, 2, 3 and 4 suggest that greater cost savings from the EM may be obtained when stronger correlations among demands in different periods exist. However, this does not apply to all the cases. Figures 5 and 6 show such examples.

Figures 5 and 6 show that cost savings become smaller when d_0 becomes closer to the demand process mean ($d_0 = 100$), while greater cost savings from EM can be obtained when d_0 deviates greatly from the demand process mean ($d_0 = 50/150$). Furthermore, the cost savings from EM decrease when N increases. Non-symmetric cost savings between $\mathrm{d}_0 = 50$ and $\mathrm{d}_0 = 150$ are due to the following two reasons: (1) the unit salvage cost ($h = 1$) is not equal to the unit stockout cost ($\pi = 5$) and (2) demand fluctuation quickly diminishes and the cost savings are the greatest in the first few periods. When $d_0 = 50(150)$, demand is low (high) in the first few periods. Thus, transactions in the EM greatly reduce the inventory salvage (stockout) cost. Since the unit salvage cost ($h = 1$) is not equal to the unit stockout cost ($\pi = 5$), we observe that the reduction in the inventory salvage cost is much smaller than the reduction in the stockout cost.

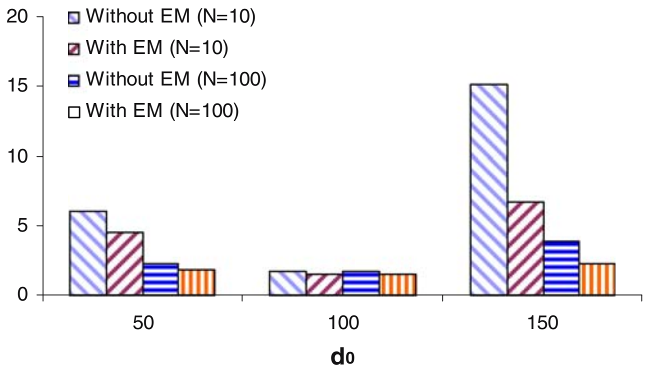

Fig. 5 Average cost per period when $\phi = 0.5, \sigma = 1$

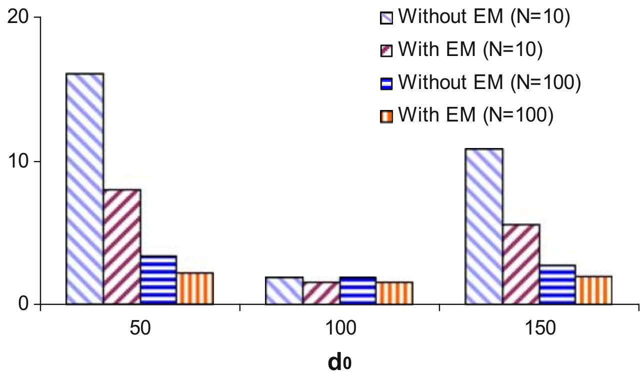

Fig. 6 Average cost per period when $\phi = -0.5, \sigma = 1$

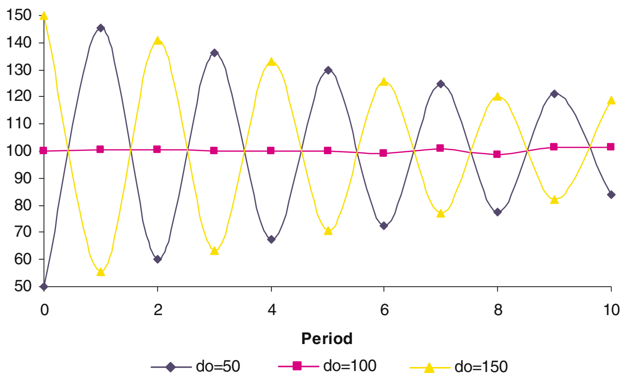

Fig. 7 Sample demands when $N = 10, \phi = -0.9, \sigma = 1$

In order to further analyze the results, sample demands under different d_0 and N are plotted in Figs. 7, 8, 9 and 10. Figures 7 and 8 show that demands fluctuate greatly in different periods ($N = 10$) when d_0 deviates greatly from

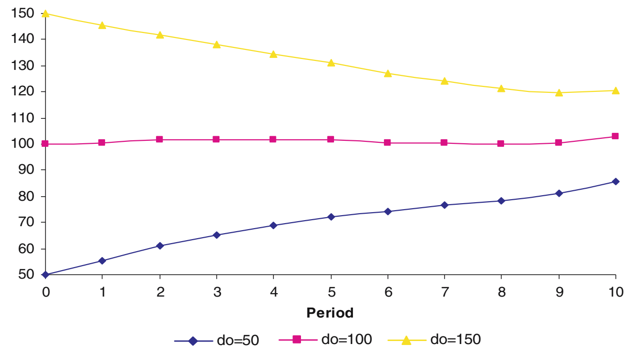

Fig. 8 Sample demands when $N = 10, \phi = 0.9, \sigma = 1$

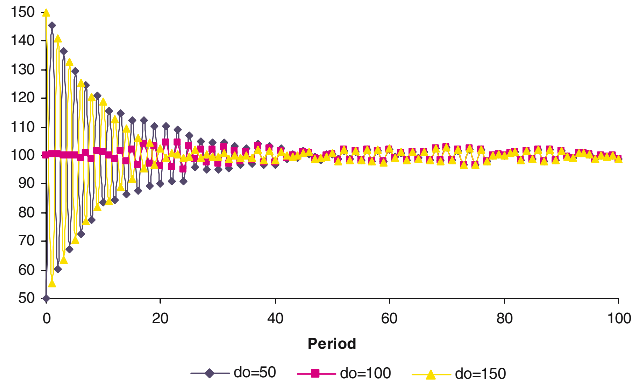

Fig. 9 Sample demands when $N = 100, \phi = -0.9, \sigma = 1$

the demand process mean ($d_0 = 50/150$). When there are greater correlations among demands in different periods, the demand forecast in the current period can be updated by taking into account demands in previous periods. Using this updated demand forecast, the retailer can then adjust her inventory level by purchasing and selling products in the EM. This results in significant cost savings. On the contrary, fluctuation in the demands becomes smaller when $d_0 = 100$. Thus, the cost savings from the EM also become smaller.

Figures 9 and 10 illustrate sample demands when $N = 100$. Figures 7, 8, 9 and 10 show that regardless of d_0, demand fluctuation quickly diminishes and the cost savings from EM are reduced accordingly. Thus, the average cost savings over N periods decrease as N increases, as shown in Figs. 1, 2, 3, 4, 5 and 6.

However, it is noticeable that the cost savings from EM do not disappear even when the variation of the demand distribution in each period becomes very small. This is because the forecasted demand in period $t + 1$ is to follow the Normal distribution with a mean of $\mu + \phi^*(d_t - \mu)$ and a variance of σ^2,

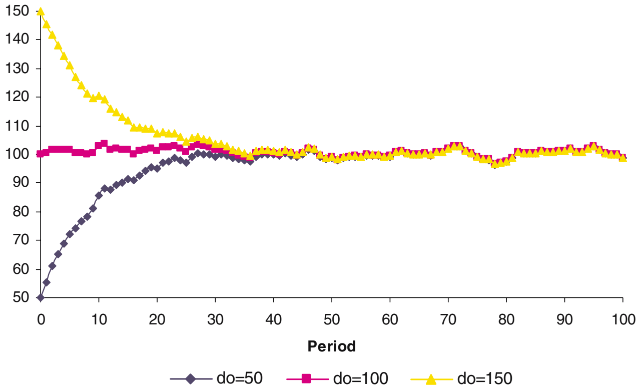

Fig. 10 Sample demands when $N = 100, \phi = 0.9, \sigma = 1$

where $\mu + \phi^*(d_t - \mu)$ changes in each period due to the stochastic demand d_t. Thus, cost reduction is obtained by employing the EM, based on this stochastic demand.

With EM which allows the retailer to purchase the exact amount to satisfy its demand, we are able to consider up-to-date demand information to forecast demand for the next period, and this is where the cost savings are produced (without EM, we may only use the mean demand, when purchasing a fixed quantity for a fixed time period). Thus, the accurate demand forecast is important to maximize the cost savings.

When d_0 is near the mean demand, the demand forecast model will help to predict the next period demand given that the autocorrelation is not zero (the variance of the forecasted demand will be smaller though). However, when d_0 is well away from the mean demand, the demand forecast model plays an even greater role, as it is able to predict the next period demand and the next period demand will be well away from the mean demand. That is why the cost savings become even larger when d_0 is well away from the mean demand. While a larger σ value would dampen out the impact of d_0, a large σ may easily lead to a negative demand level and thus, moderate values of σ are employed in this study.

In conclusion, when both (1) demands fluctuate greatly in different periods and (2) there are strong correlations among demands in different periods, greater cost savings can be obtained from EM.

5 Summary

This paper studies an inventory control problem in the presence of an EM over a finite horizon with auto-correlated demand. At the beginning of each period, a fixed quantity of order is received from the supplier, where the quantity must be determined first and cannot be changed thereafter. Inventory level can then be adjusted at each period by purchasing and selling products in an EM. The available supply and demand quantities in the EM depend on the prices offered. The retailer's optimal purchasing and selling quantities, and respective prices in the EM are computed, and the expected total cost is shown to be convex with respect to the order quantity from the supplier. Numerical experiments are conducted to measure the impacts of the demand processes on the cost savings from the EM. It is found that when demands fluctuate greatly in different periods and there are strong correlations among demands in different periods, greater cost savings can be obtained by purchasing and selling products in the EM to adjust inventory levels.

The assumption that the product is perishable with the lifetime of one period may cause some loss of generality for the theoretic results obtained in Sect. 3. For products with the lifetime of longer than one period, still the bidding decision proposed in this study can be employed as a heuristic solution. Moreover, the insights obtained from this numerical experiment can also apply to these products. A future research topic is to consider the EM where demand and supply are random variables, following a known distribution. Under the assumption, the mean and the variance are a deterministic function of purchasing and selling prices.

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Appendix A: Proof of Theorem 1

In order to prove Theorem 1, the following Lemma S1 must first be proved.

Lemma S1 For $P1$, when $Q < X_k$, an upper bound for $C^*_{p,k}$ is $C^u_{p,k}$, satisfying $C^u_{p,k} + L'_k\left(Q + \gamma(C^u_{p,k})\right) = 0$. When $Q > X_k$, an upper bound for $C^*_{s,k}$ is $C^u_{s,k}$, satisfying $C^u_{s,k} - L'_k\left(Q - \gamma(C^u_{s,k})\right) = 0$.

Proof It is obvious that when $Q < X_k$, $C^*_{p,k}$ and $Y^*_{p,k}$ must satisfy $C^*_{p,k} + L'_k\left(Q + Y^*_{p,k}\right) = 0$. Therefore, when $C^*_{p,k} > C^u_{p,k}$, $Y^*_{p,k} < \gamma(C^u_{p,k}) < \gamma(C^*_{p,k})$. This contradicts Lemma 1. Therefore, an upper bound for $C^*_{p,k}$ is $C^u_{p,k}$, satisfying $C^u_{p,k} + L'_k\left(Q + \gamma(C^u_{p,k})\right) = 0$. Similarly, when $Q > X_k$, an upper bound for $C^*_{s,k}$ is $C^u_{s,k}$, satisfying $C^u_{s,k} - L'_k\left(Q - \gamma(C^u_{s,k})\right) = 0$. □

Suppose that $Q < X_k$. From Lemma 1, $C^*_{s,k} = 0$ and $Y^*_{p,k} = \gamma(C^*_{p,k})$. In addition, Lemma S1 indicates that $0 \le C^*_{p,k} \le C^u_{p,k}$. Therefore, $C^*_{p,k}$ is obtained as follows:

$$A_k(C_{p,k}, C_{s,k}, Q|\hat{D}_{k-1}) = C^*_{p,k}\gamma(C_{p,k}) + C^*_{s,k}\eta(C_{s,k}) + L_k\big(Q + \gamma(C_{p,k}) - \eta(C_{s,k})\big)$$

$$\underset{0\le C_{p,k}\le C^u_{p,k}}{\mathrm{Min}} : A(C_{p,k}, 0, Q|\hat{D}_{k-1}) = C_{p,k} * \gamma(C_{p,k}) + L_k\big(Q + \gamma(C_{p,k})\big) \tag{a.1}$$

By taking the first and second order derivatives of $A(C_{p,k}, 0, Q|\hat{D}_{k-1})$:

$$A'(C_{p,k}, 0, Q|\hat{D}_{k-1}) = \gamma(C_{p,k}) + \Big[C_{p,k} + L_k'\big(Q + \gamma(C_{p,k})\big)\Big] * \gamma'(C_{p,k})$$

$$A''(C_{p,k}, 0, Q|\hat{D}_{k-1}) = 2 * \gamma'(C_{p,k}) + L_k''\big(Q + \gamma(C_{p,k})\big) * \gamma'(C_{p,k})^2 + \Big[C_{p,k} + L_k'\big(Q + \gamma(C_{p,k})\big)\Big] * \gamma''(C_{p,k})$$

Because $\gamma(C_{p,k})$ is a concave increasing function of $C_{p,k}, \gamma'(C_{p,k}) > 0$ and $\gamma''(C_{p,k}) < 0$.

Since $C_{p,k} + L_k'\big(Q + \gamma(C_{p,k})\big) = 0$ is an increasing function of $C_{p,k}$ and $C^u_{p,k} + L_k'\Big(Q + \gamma(C^u_{p,k})\Big) = 0$ (from Lemma S1), $C_{p,k} + L'\big(Q + \gamma(C_{p,k})\big) \le 0$ from $0 \le C_{p,k} \le C^u_{p,k}$ (directly follows Eq. (a.1)).

Finally, $L_k''\big(Q + \gamma(C_{p,k})\big) = (\pi + h) * f_k\Big(Q + \gamma(C_{p,k})\big|\,\hat{D}_{k-1}\Big) \ge 0$.

Therefore, $A''(C_{p,k}, 0, Q|\hat{D}_{k-1}) > 0$ and $A(C_{p,k}, 0, Q|\hat{D}_{k-1})$ is convex with respect to $C_{p,k}$. $C^*_{p,k}$ can be obtained by solving $A'(C_{p,k}, 0, Q|\hat{D}_{k-1}) = 0$. Similarly, when $Q{>}X_k, A(0, C_{s,k}, Q|\hat{D}_{k-1})$ is convex in $C_{s,k}$, and $C^*_{s,k}$ can be computed accordingly. □

Appendix B: Proof of Theorem 2

In order to prove Theorem 2, it is only necessary to prove that $C_k(Q|\hat{D}_{k-1})$ is convex with respect to Q, given that $C^*_{p,k}, C^*_{s,k}, Y^*_{p,k}$ and $Y^*_{s,k}$ are obtained by Theorem 1 and Lemma 1.

$$C_k(Q|\hat{D}_{k-1}) = \underset{0\le Y_{p,k}, 0\le Y_{s,k}\le Q+Y_{p,k}}{\mathrm{Min}} : B_k(Y_{p,k}, Y_{s,k}, Q|\hat{D}_{k-1}) \tag{b.1}$$

$$B_k(Y_{p,k}, Y_{s,k}, Q|\hat{D}_{k-1}) = Y_{p,k} * \gamma^{-1}(Y_{p,k}) + Y_{s,k} * \eta^{-1}(Y_{s,k}) + L_k\big(Q + Y_{p,k} - Y_{s,k}\big)$$

Since both $\gamma(.)$ and $\eta(.)$ are concave increasing functions and $L_k(.)$ is a convex function, it is obvious that $B_k(Y_{p,k}, Y_{s,k}, Q|\hat{D}_{k-1})$ is convex with respect to $Y_{p,k}, Y_{s,k}$ and Q. From this property and Eq. (b.1), it is straightforward that $C_k(Q|\hat{D}_{k-1})$ is convex in Q. [See the results on convex functions in Chap. 5 of Rockafellar (1970)] $\square$

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